

MEMRISTORS

Tea talk

13 May 2014

WHAT'S A MEMRISTOR?

- Memory-resistor:
the missing 4th circuit element
- predicted by Leon Chua (1971)
Einstein of Circuit Theory?
- discovered by HP guys (2008)
- a big hype - we can now do a lot more with circuits! Really? (such as building brain-like computers!?)

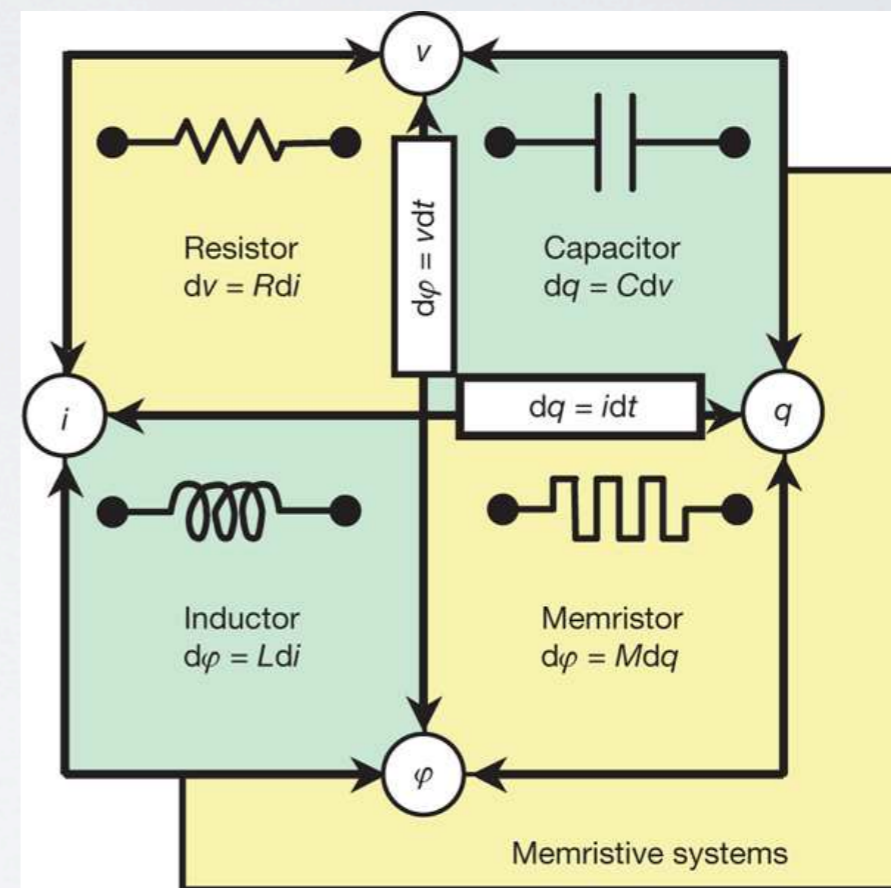


FIG. 1: Figure adapted from *Nature*.

4 BASIC CIRCUIT VARIABLES

- voltage v
- current i
- charge q
- magnetic flux ϕ

$$q = \int i \, dt \quad \phi = \int v \, dt$$

CIRCUIT ELEMENTS

- A circuit element is:

any device that imposes a relationship between two circuit variables

- Resistor: $v - i$

- Capacitor: $q - v$

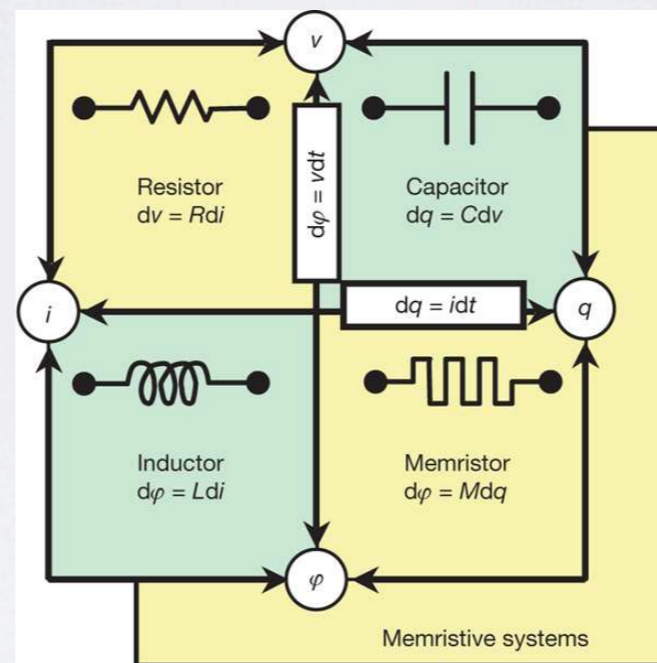
- Inductor: $\phi - i$

4 BASIC CIRCUIT VARIABLES

4 FUNDAMENTAL CIRCUIT ELEMENTS?

- voltage
- current
- charge
- magnetic flux

v
 i
 q
 ϕ



- Resistor: $v - i$
- Capacitor: $q - v$
- Inductor: $\phi - i$

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$$q = \int i dt \quad \phi = \int v dt$$

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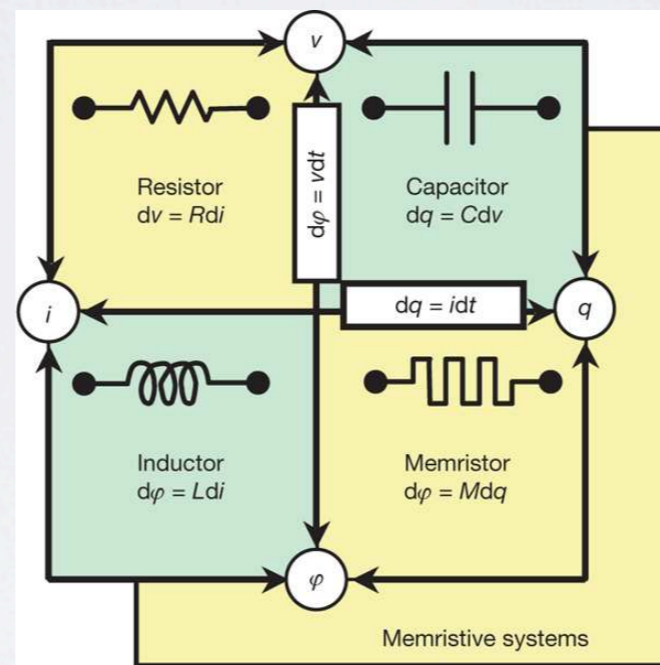


FIG. 1: Figure adapted from *Nature*.

- Resistor: $v - i$
- Capacitor: $q - v$
- Inductor: $\phi - i$
- Memristor: $\phi - q$

$$q = \int i dt \quad \phi = \int v dt$$

4 BASIC CIRCUIT VARIABLES

4 FUNDAMENTAL CIRCUIT ELEMENTS?

- voltage
- current
- charge
- magnetic flux

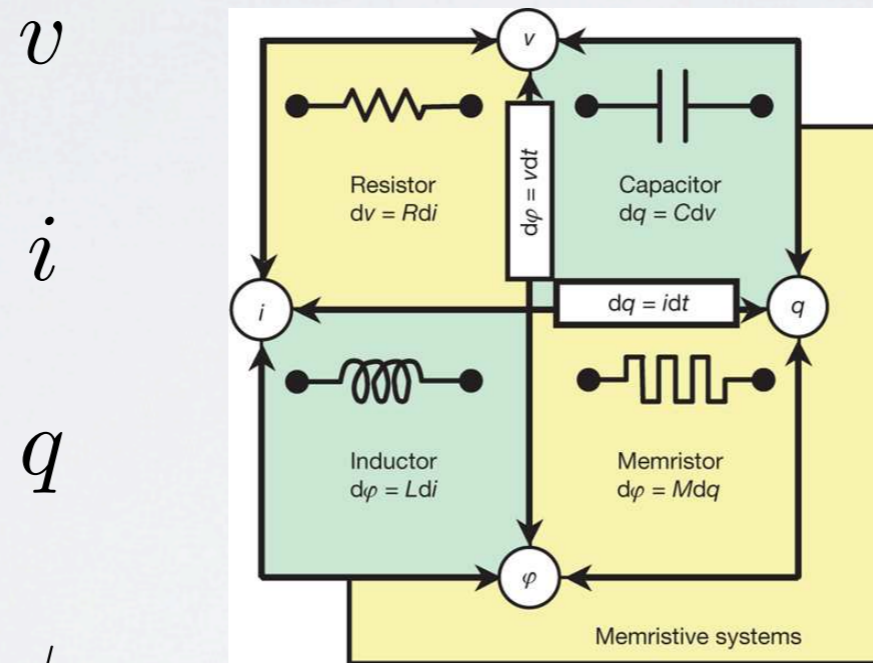


FIG. 1: Figure adapted from *Nature*.

$$R = \frac{dv}{di}$$

$$C = \frac{dq}{dv}$$

$$L = \frac{d\phi}{di}$$

$$M = \frac{d\phi}{dq}$$

$$q = \int i dt \quad \phi = \int v dt$$

4 ELEMENT THEORY

- Chua

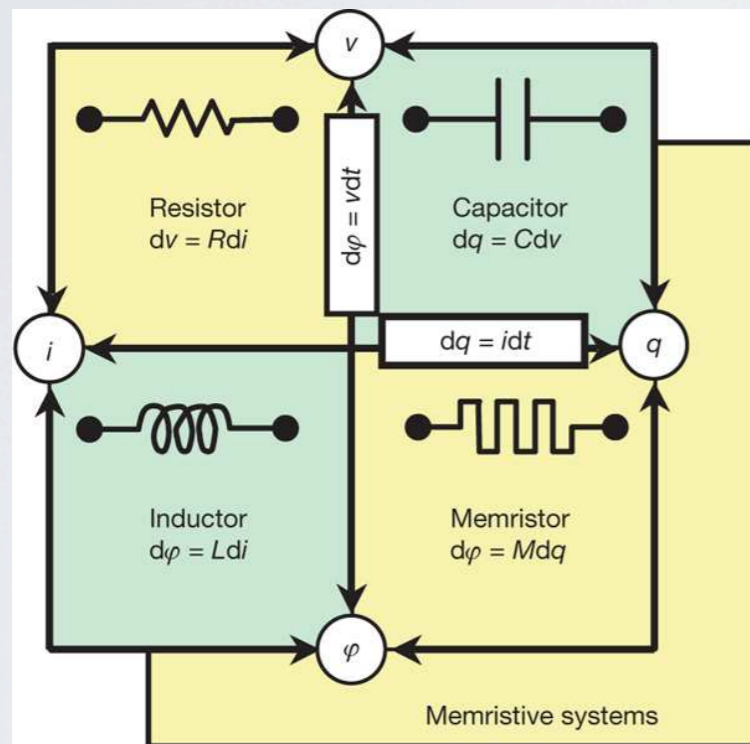


FIG. 1: Figure adapted from *Nature*.

- Aristotle



MEMRISTOR IS FUNDAMENTAL

- Because M cannot be replicated by combining other passive elements: R,L,C
- Transistor is not fundamental
- R,L,C are LTI (Linear-time-invariant). M is not.

- but M can be replicated with active elements (amplifier)

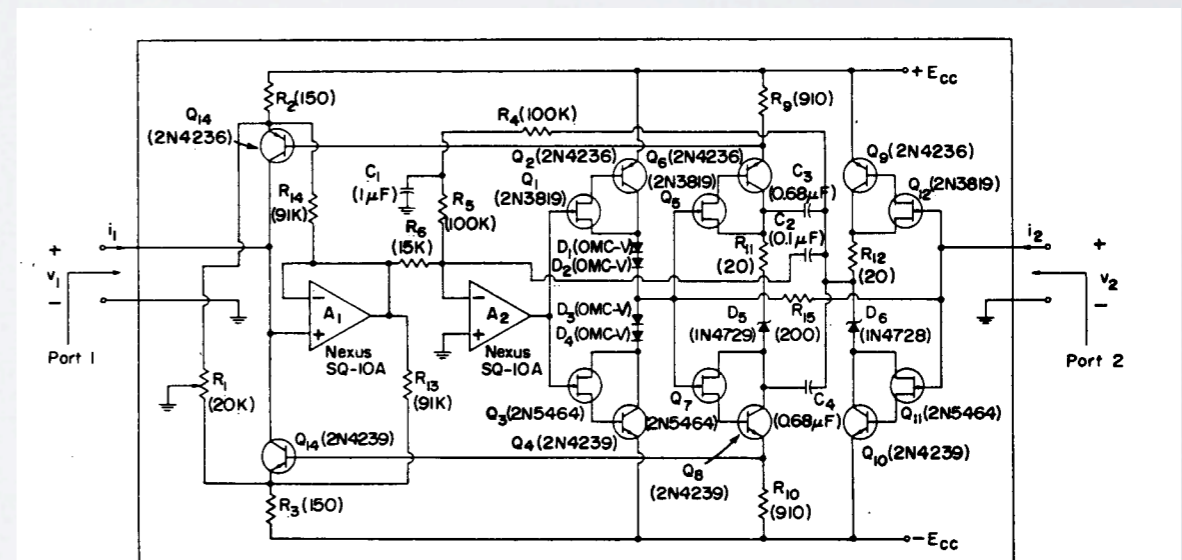


Fig. 2. Practical active circuit realization of type-1 M - R mutator based on realization 1 of Table I.

MEMRISTOR

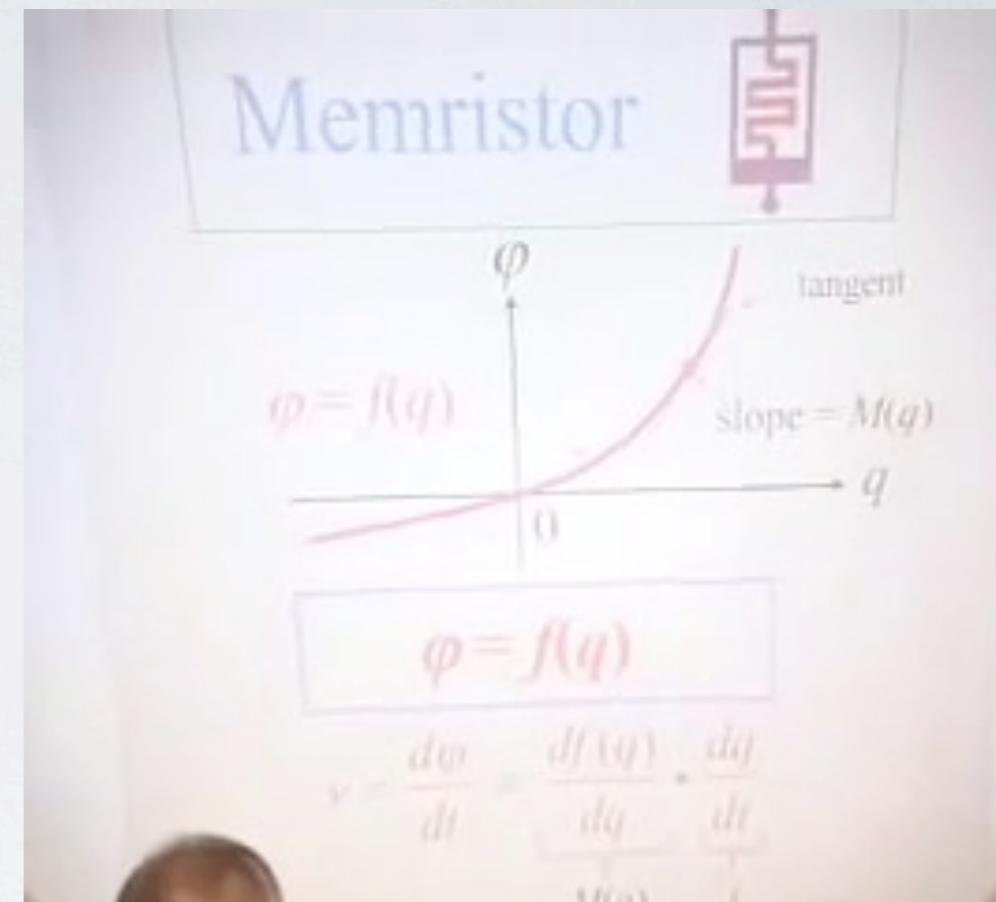
- Charge - Flux relationship

$$\phi = f(q)$$

$$\frac{d\phi}{dt} = M(q) \frac{dq}{dt}$$

$$M(q) = f'(q)$$

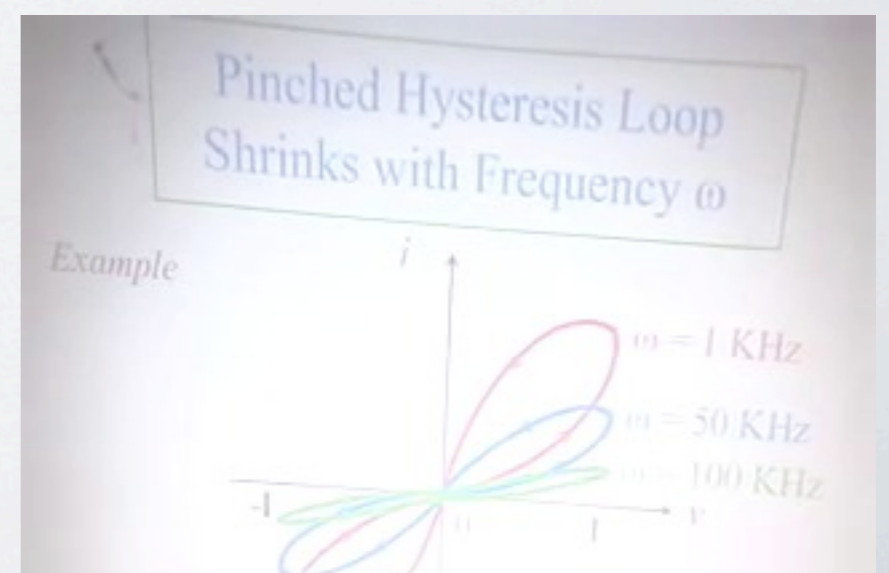
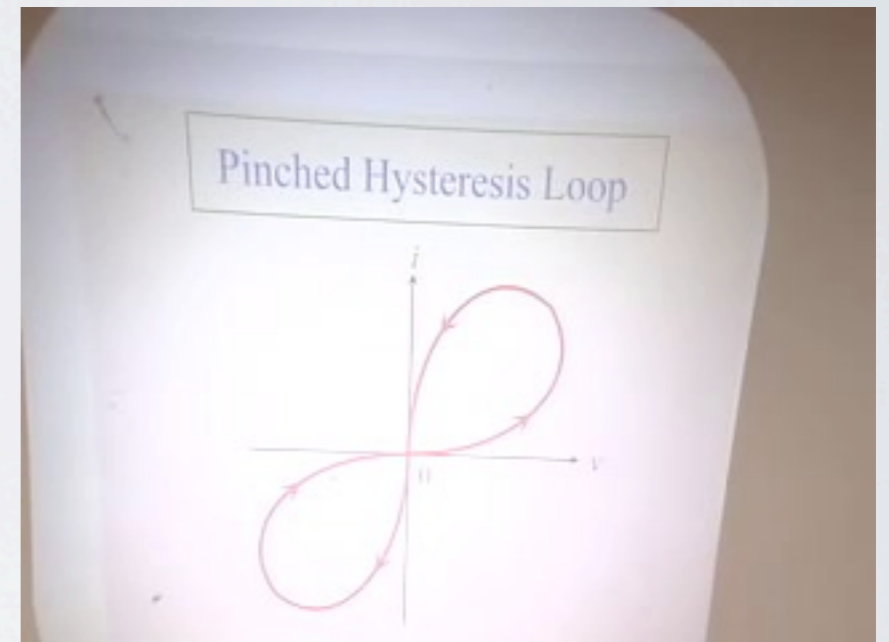
$$v = M(q) i$$



$$M = \frac{d\phi}{dq} = \frac{d\phi/dt}{dq/dt} = \frac{v}{i}$$

EXPERIMENTAL CHECK

- You have a memristor if&f
 - it exhibits a pinched hysteresis loop,
 - the loop shrinks with freq, and
 - it shrinks to a straight line



MECHANICAL MEMRISTOR

A Mechanical Memristor

$p = \int_{-\infty}^t f(\tau) d\tau$
 $p = f(x)$
 $x = \int_{-\infty}^t v(\tau) d\tau$

$p = \text{momentum}, \quad f = \text{force}$
 $x = \text{displacement (distance)}$
 from some reference point, $v = \text{velocity}$

Constitutive Relation: $p = f(x)$

$\frac{dp}{dt} = \frac{df(x)}{dx} \frac{dx}{dt}$

Force = Momentum / Time

- just for illustration

Mechanical Resistor

Dashpot / Cylinder
 Fixed metal disc for pivoting the piston rod
 Force $f(t)$
 distance $x(t)$
 $x = 0$
 $x = D$
 $\text{velocity } v(t) = \frac{dx}{dt}$
 $f = R v$
 $R = \text{Mechanical res. due to friction}$

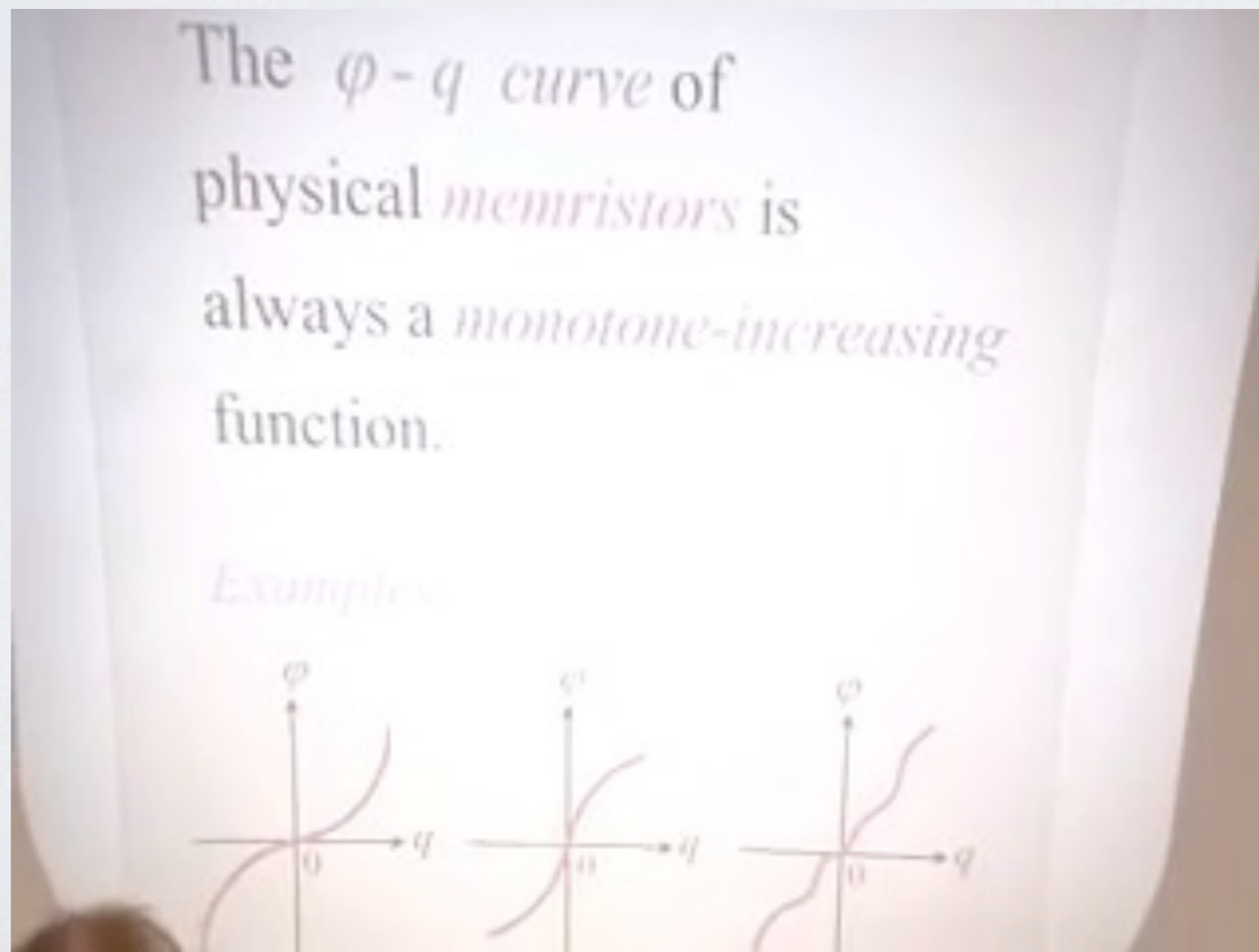
Mechanical Memristor

Dashpot / Cylinder
 Fixed metal disc for pivoting the piston rod
 Force $f(t)$
 distance $x(t)$
 $x = 0$
 $x = D$
 $p = \int_{-\infty}^t f(\tau) d\tau$
 $f = M(x) v$

THEOREM

- PASSIVE MEMRISTOR IS FEASIBLE

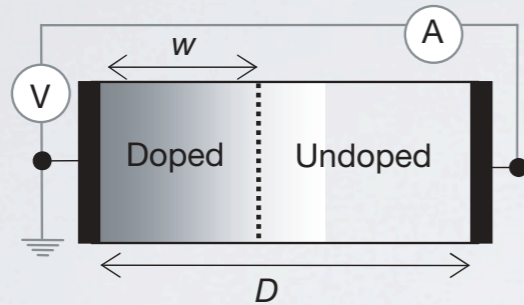
- IF&F



The missing memristor found

Dmitri B. Strukov¹, Gregory S. Snider¹, Duncan R. Stewart¹ & R. Stanley Williams¹

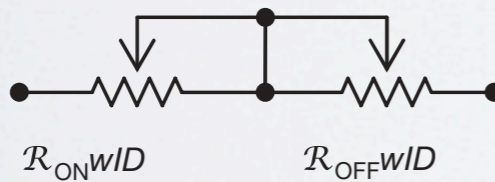
- Tungsten **a**xide semiconductor



Undoped:



Doped:

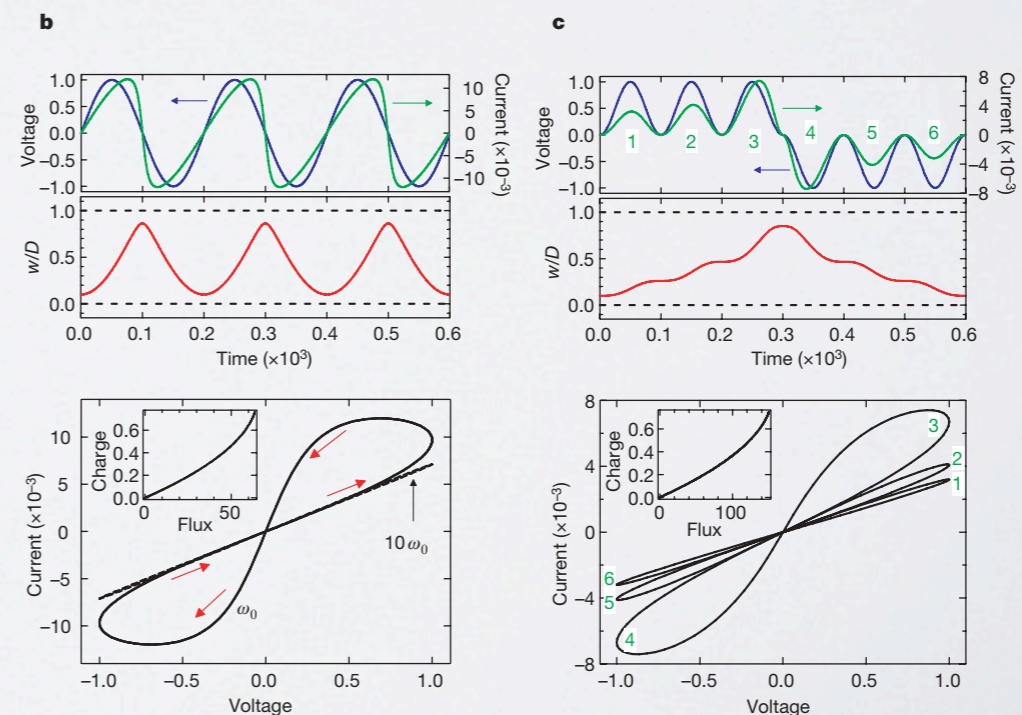


$$v(t) = \left(\mathcal{R}_{\text{ON}} \frac{w(t)}{D} + \mathcal{R}_{\text{OFF}} \left(1 - \frac{w(t)}{D} \right) \right) i(t)$$

$$\frac{dw(t)}{dt} = \mu_V \frac{\mathcal{R}_{\text{ON}}}{D} i(t)$$

$$w(t) = \mu_V \frac{\mathcal{R}_{\text{ON}}}{D} q(t)$$

$$M(q) = \mathcal{R}_{\text{OFF}} \left(1 - \frac{\mu_V \mathcal{R}_{\text{ON}}}{D^2} q(t) \right)$$



GENERALIZATION: MEMRISTIVE SYSTEMS

- any resistive system with internal state variables is memristive

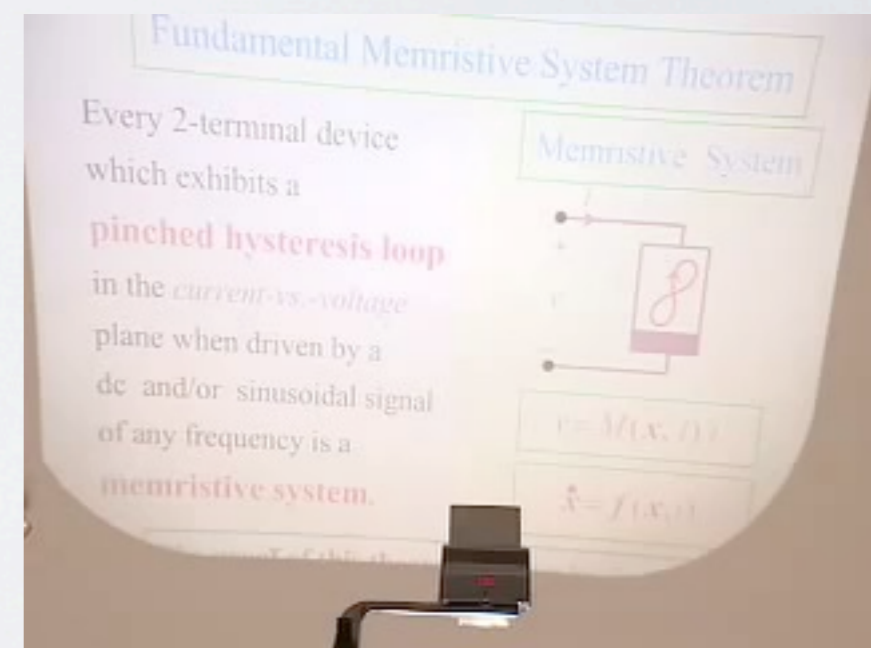
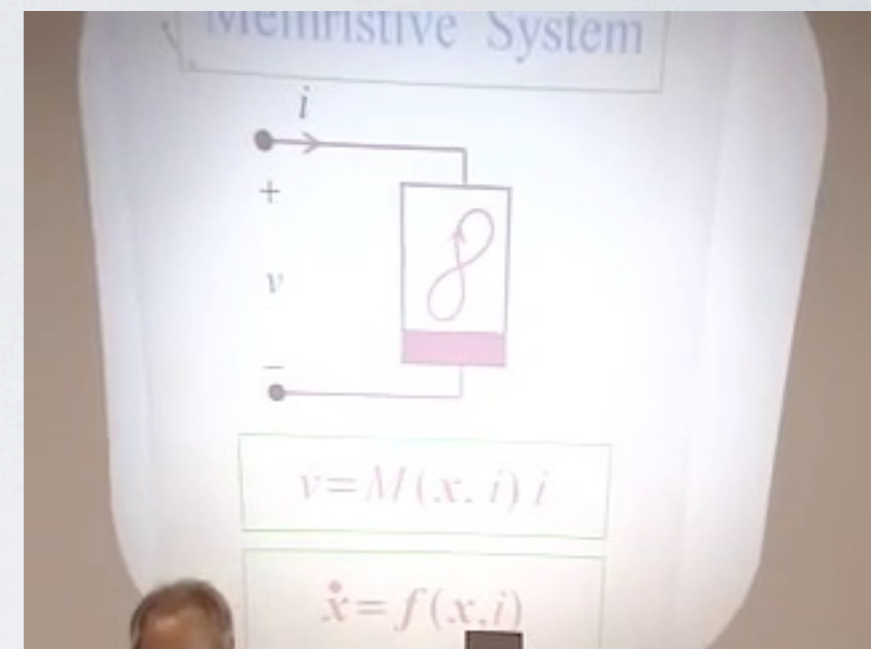
$$v(t) = M(\vec{x}, i) i(t) \quad i(t) = M(\vec{x}, v)^{-1} v(t)$$

$$\dot{x} = g(\vec{x}, i) \quad \dot{x} = g(\vec{x}, v)$$

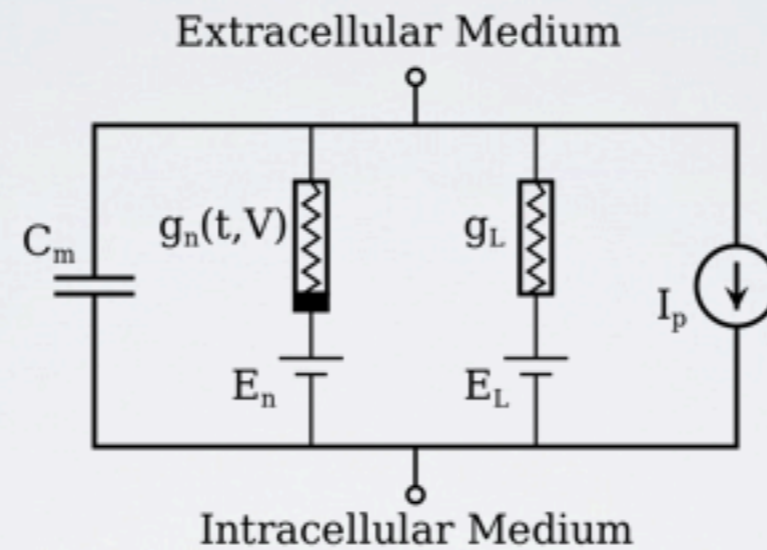
current controlled or voltage controlled

- e.g. thermistor ($x = \text{temperature}$)

- Theorem:** every system with a pinched hysteresis is memristive



HH NEURON MODEL IS MEMRISTIVE or memductive



$$I = C_m \frac{dV_m}{dt} + \bar{g}_K n^4 (V_m - V_K) + \bar{g}_{Na} m^3 h (V_m - V_{Na}) + \bar{g}_l (V_m - V_l),$$

$$\frac{dn}{dt} = \alpha_n(V_m)(1 - n) - \beta_n(V_m)n$$

$$\frac{dm}{dt} = \alpha_m(V_m)(1 - m) - \beta_m(V_m)m$$

$$\frac{dh}{dt} = \alpha_h(V_m)(1 - h) - \beta_h(V_m)h$$

APPLICATIONS

Neural Net Application:

Neuromorphic engineering of synapses <http://www.artificialbrains.com/darpa-synapse-program#memristor-chip>

Williams' solid-state memristors can be combined into devices called [crossbar latches](#), which could replace transistors in future computers, given their much higher circuit density.

They can potentially be fashioned into [non-volatile](#) solid-state memory, which would allow greater data density than hard drives with access times similar to [DRAM](#), replacing both components.[\[56\]](#) HP prototyped a crossbar latch memory that can fit 100 [gigabits](#) in a square centimeter,[\[9\]](#) and proposed a scalable 3D design (consisting of up to 1000 layers or 1 [petabit](#) per cm³).[\[57\]](#) In May 2008 HP reported that its device reaches currently about one-tenth the speed of DRAM.[\[58\]](#) The devices' resistance would be read with [alternating current](#) so that the stored value would not be affected.[\[59\]](#) In May 2012 it was reported that access time had been improved to 90 nanoseconds if not faster, approximately one hundred times faster than contemporaneous flash memory, while using one percent as much energy.[\[60\]](#)

Memristor patents include applications in [programmable logic](#),[\[61\]](#) [signal processing](#),[\[62\]](#) [neural networks](#),[\[63\]](#) [control systems](#),[\[64\]](#) [reconfigurable computing](#),[\[65\]](#) [brain-computer interfaces](#)[\[66\]](#) and [RFID](#).[\[67\]](#) Memristive devices are potentially used for stateful logic implication, allowing a replacement for CMOS-based logic computation. Several early works in this direction are reported.[\[68\]](#) [\[69\]](#)

Memristor definition and criticism

According to the original 1971 definition, the memristor was the fourth fundamental circuit element, forming a non-linear relationship between electric charge and magnetic flux linkage. In 2011 [Chua](#) argued for a broader definition that included all 2-terminal non-volatile memory devices based on resistance switching. Williams argued that [MRAM](#), [phase change memory](#) and [RRAM](#) were memristor technologies. Some researchers argued that biological structures such as blood and skin fit the definition. Others argued that the memory device under development by [HP Labs](#) and other forms of [RRAM](#) were not memristors but rather part of a broader class of variable resistance systems [\[19\]](#) and that a broader definition of memristor is a scientifically unjustifiable land grab that favored HP's memristor patents.[\[20\]](#)

Meuffels and Schroeder noted that one of the early memristor papers included a mistaken assumption regarding ionic conduction.[\[21\]](#) Meuffels and Soni discussed issues and problems in the realization of memristors.[\[4\]](#) They claimed that the physics behind the HP memristor model conflicts with fundamentals of solid state electrochemistry as the coupling of electronic/ionic diffusion currents was not considered. Additionally, they pointed to issues concerning fundamentals of [non-equilibrium thermodynamics](#): the dynamic state equations set up for memristors like the HP memristor imply the possibility of violating [Landauer's principle](#) of the minimum amount of energy required to change "information" states in a system.[\[4\]](#) This critique was endorsed by [Di Ventra](#) and Pershin.[\[5\]](#)

Nonvolatile information storage requires the existence of energy barriers that separate distinct memory states from each other.[\[4\]\[5\]](#) Memristors whose resistance (memory) states depend only on the current (like the HP memristor) or voltage history would be unable to protect their memory states against unavoidable fluctuations and thus permanently suffer information loss: the proposed hypothetical concept provides no physical mechanism enabling such systems to retain memory states after the applied current or voltage stress is removed. Such elements can therefore not exist, as they would always be susceptible to a so-called "stochastic catastrophe".[\[5\]](#)

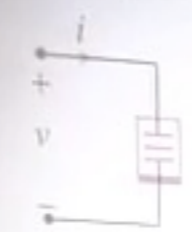
Other researchers noted that memristor models based on the assumption of linear ionic drift do not account for asymmetry between set time (high-to-low resistance switching) and reset time (low-to-high resistance switching) and do not provide ionic mobility values consistent with experimental data. Non-linear ionic drift models have been proposed to compensate for this deficiency.[\[22\]](#)

Yet another article from researchers of [ReRAM](#) concluded that "If the basic equations do not reflect the actual device physics well, as we see for the basic memristor equations, with or without window functions, low predictivity is given..".[\[23\]](#)

Martin Reynolds, an electrical engineering analyst with research outfit [Gartner](#), commented that while HP was being sloppy in calling their device a memristor, critics were being pedantic in saying it was not a memristor.[\[24\]](#)

MEM-CAPACITORS, MEM-INDUCTORS..

Memory Capacitor

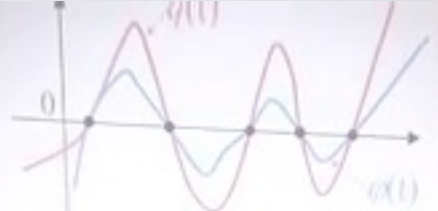


$\sigma \triangleq \int_{-\infty}^t q(t) dt$
 $\sigma = f(\varphi)$
 $\varphi \triangleq \int_{-\infty}^t v(t) dt$
 $\frac{d\sigma}{dt} = \frac{dq(t)}{dt} = \frac{df(\varphi)}{d\varphi} \frac{d\varphi}{dt}$
 $\frac{d\sigma}{dq} = \frac{df(\varphi)}{d\varphi} \frac{1}{v}$
 $C(\varphi) = \frac{df(\varphi)}{d\varphi}$


Constitutive Relation

$\sigma = f(\varphi) \Rightarrow q = C(\varphi) v$

where $C(\varphi) \triangleq \frac{df(\varphi)}{d\varphi}$, Farads



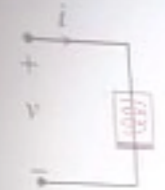
2. The q -vs.- φ loci is a *pinched hysteresis loop*.



Theorem

The *memory capacitor* is a *lossless element*

Memory Inductor



$\rho \triangleq \int_{-\infty}^t \varphi(t) dt$
 $\rho = f(q)$
 $q \triangleq \int_{-\infty}^t i(t) dt$
 $\frac{d\rho}{dt} = \frac{d\varphi(t)}{dt} = \frac{df(q)}{dq} \frac{dq}{dt}$
 $\frac{d\rho}{dq} = \frac{df(q)}{dq} \frac{1}{i}$
 $L(q) = \frac{df(q)}{dq}$

Constitutive Relation

$\rho = f(q) \Rightarrow \varphi = L(q) i$

where $L(q) \triangleq \frac{df(q)}{dq}$, Henrys

is called the *memory inductance*.

THERE'S MORE

The First 25 Circuit Elements

Four fundamental circuit elements

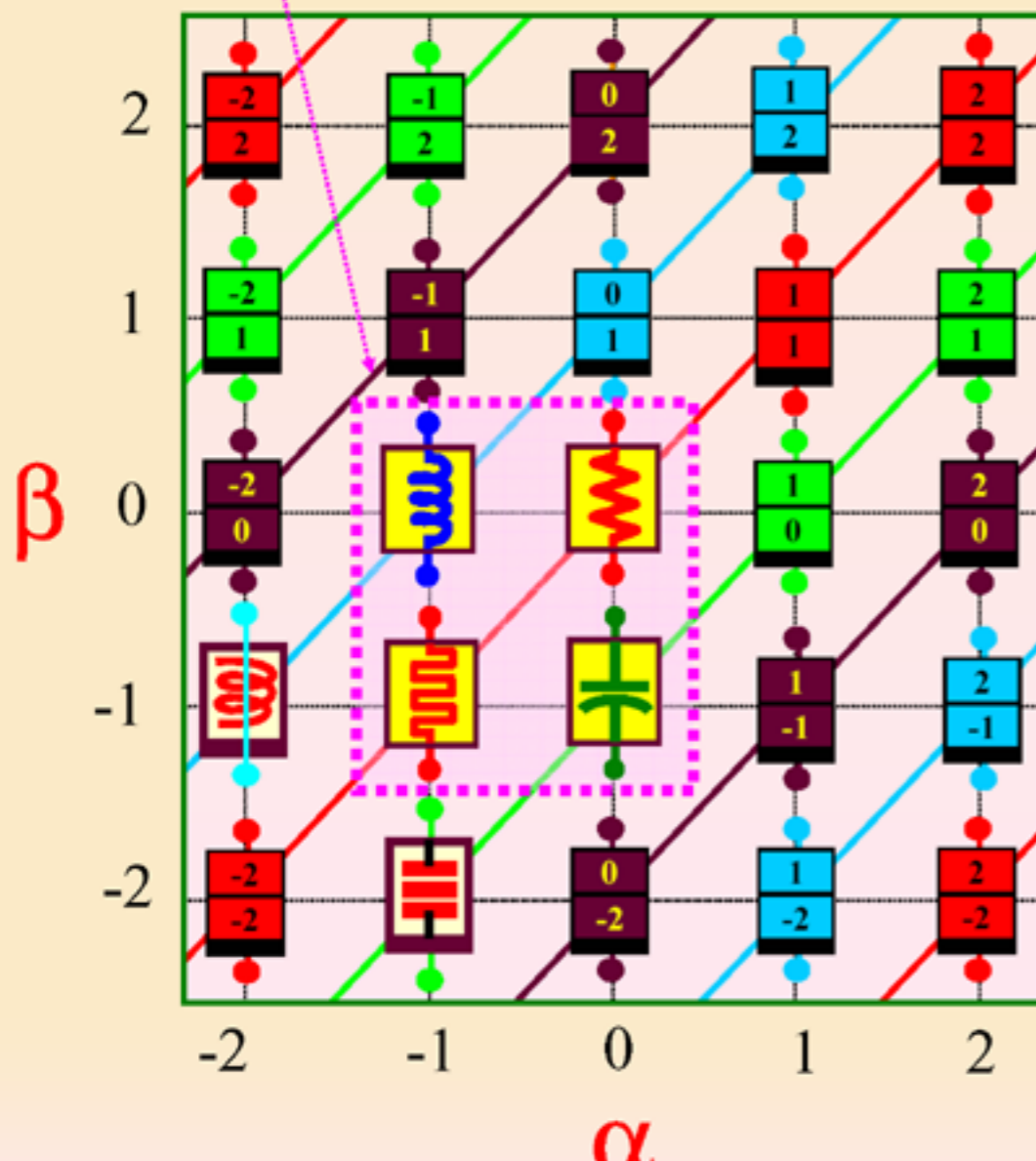


Fig. 11 An enlargement of the first 25 axiomatically defined circuit elements from the periodic table of circuit elements (Fig. 31 of [3]) where the four basic circuit elements (resistor, capacitor, inductors and memristor) are replaced by their symbols. The *memcapacitor* is located at $(a, b) = (-1, -2)$ and the *meminductor* is located at $(a, b) = (-2, -1)$. Observe that since these two elements require double time integrals of voltage and current, their dynamics are of a higher order than those of the four basic circuit elements enclosed inside the dotted *red box*

REFERENCES

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L Chua - **Circuit** Theory, IEEE Transactions on, 1971
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- **The missing memristor found**

DB Strukov, GS Snider, DR Stewart, RS Williams - Nature, 2008
<http://www.ece.ucsb.edu/~strukov/papers/2008/Nature2008.pdf>

- **Memristor for Introductory Physics**

F Wang <http://arxiv.org/pdf/0808.0286.pdf>

- **Resistance switching memories are memristors**

Leon Chua <http://goo.gl/k9Yn5N>

- **How we found the missing memristor**

R Williams - Spectrum, IEEE, 2008 - ieeexplore.ieee.org

<http://dat175-memristor.googlecode.com/svn/trunk/Papers/MemristorSpectrum.pdf>

- Circuit elements with memory: memristors, memcapacitors and meminductors

Massimiliano Di Ventra, Yuriy V. Pershin, and Leon O. Chua, Fellow, IEEE
<http://arxiv.org/pdf/0901.3682.pdf>

Memristor and Memristive Systems Symposium

<http://youtu.be/QFdDPzcZwbs>
http://youtu.be/o9u9o_ToQwM
http://youtu.be/h7cX_m5lKxk
http://youtu.be/PSnC_6U5tKl

<http://www.artificialbrains.com/darpa-synapse-program#memristor-chip>

Aristotle's Law of Motion

Force proportional to velocity

$$F = mv$$

Newton's Second Law

For a given mass, force is proportional to acceleration

$$f = m a$$

Newton's Second Law: $F = \frac{dp}{dt}$, $p = mv$

$$\int_{-\infty}^{\infty} F(t_1) dt_1 = \int_{-\infty}^{\infty} \left[\frac{m_0}{\sqrt{(1 - v^2/c^2)^{3/2}}} \right] \frac{dv}{dt_1} dt_1$$

Aristotle's Law of Motion

$$F = m v$$

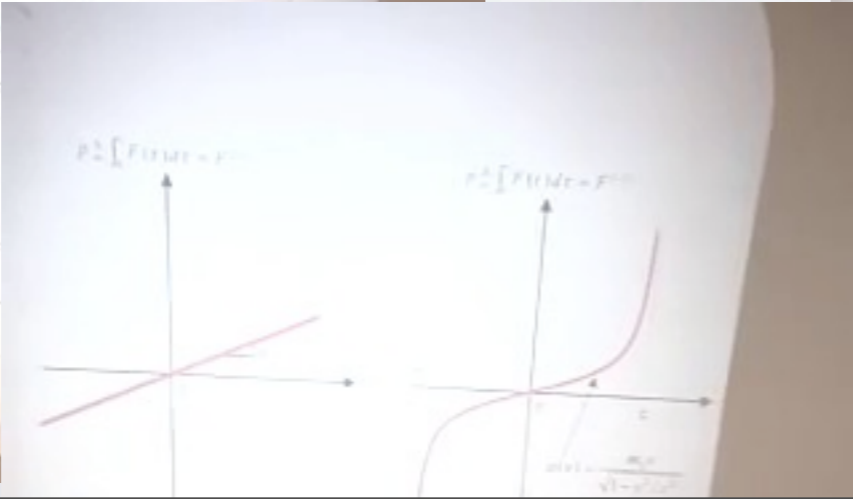
Newton's Law of Motion

$$F = m a$$

Einstein's Law of Motion

$$p = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

for $v \ll c$, $m = m_0$



Einstein's Relativistic Law of Motion

$$p = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

$$F = \frac{dp}{dt}$$

where